

**Paper Reference 9MA0/02**  
**Pearson Edexcel**  
**Level 3 GCE**

# **Mathematics**

**Advanced**

**Paper 2: Pure Mathematics 2**

**Wednesday 14 October 2020 – Afternoon**

**Time: 2 hours plus your additional time allowance.**

## **MATERIALS REQUIRED FOR EXAMINATION**

**Mathematical Formulae and Statistical Tables (Green),  
calculator**

## **ITEMS INCLUDED WITH QUESTION PAPER**

**Diagram Book**

**Answer Book**

**X66786A**

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## **INSTRUCTIONS**

**In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.**

**Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.**

**Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.**

**Do NOT write on the Question Paper.**

**You should show sufficient working to make your methods clear. Answers without working may not gain full credit.**

**Inexact answers should be given to three significant figures unless otherwise stated.**

## **INFORMATION**

**A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.**

**There are 16 questions in this Question Paper.  
The total mark for this paper is 100**

**The marks for each question are shown in brackets  
– use this as a guide as to how much time to spend on  
each question.**

## **ADVICE**

**Read each question carefully before you start to answer it.**

**Try to answer every question.**

**Check your answers if you have time at the end.**

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1. Refer to the table for Question 1 in the Diagram Book.

It shows corresponding values of  $x$  and  $y$  for

$$y = \sqrt{\frac{x}{1+x}}$$

The values of  $y$  are given to 4 significant figures.

- (a) Use the trapezium rule, with all the values of  $y$  in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} \, dx$$

giving your answer to 3 significant figures.

(3 marks)

(continued on the next page)

1. continued.

(b) Using your answer to part (a), deduce an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx$$

(1 mark)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx = 4.535$$

to 4 significant figures

(c) comment on the accuracy of your answer to part (b)

(1 mark)

(Total for Question 1 is 5 marks)

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2. Relative to a fixed origin, points **P**, **Q** and **R** have position vectors  $\underline{p}$ ,  $\underline{q}$  and  $\underline{r}$  respectively.

Given that

- **P**, **Q** and **R** lie on a straight line
- **Q** lies one third of the way from **P** to **R**

show that

$$\underline{q} = \frac{1}{3} (\underline{r} + 2\underline{p})$$

(Total for Question 2 is 3 marks)

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3. (a) Given that

$$2\log(4 - x) = \log(x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3 marks)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is NOT a solution of

$$2\log(4 - x) = \log(x + 8)$$

giving a reason for your answer.

(2 marks)

(Total for Question 3 is 5 marks)

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4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of  $x^4$  is 15 120

Find the value of  $a$

(Total for Question 4 is 3 marks)

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5. The curve with equation  $y = 3 \times 2^x$  meets the curve with equation  $y = 15 - 2^{x+1}$  at the point P

Find, using algebra, the exact  $x$  coordinate of P

(Total for Question 5 is 4 marks)

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6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2}$$

$$x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants **A**, **B** and **C**  
(3 marks)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx$$

giving your answer in the form **a + b ln 2**  
where **a** and **b** are integers to be found.  
(4 marks)

(Total for Question 6 is 7 marks)

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7. Refer to the diagram for Question 7 in the Diagram Book.

It shows a sketch of the curve **C** with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \quad x > 0$$

- (a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4 marks)

The point **P**, shown in the diagram, is the minimum turning point on **C**

- (b) Show that the **x** coordinate of **P** is a solution of

$$x = \left( \frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}}$$

(3 marks)

(continued on the next page)

Turn over

7. continued.

(c) Use the iteration formula

$$x_{n+1} = \left( \frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find

- (i) the value of  $x_2$  to 5 decimal places,
- (ii) the  $x$  coordinate of  $P$  to 5 decimal places.

(3 marks)

(Total for Question 7 is 10 marks)

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8. A curve **C** has equation  $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$  where  $a$  is a constant
- the  $y$  intercept of **C** is  $-12$
- $(x + 4)$  is a factor of  $f(x)$

find, in simplest form,  $f(x)$

(Total for Question 8 is 6 marks)

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9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol,  $\theta^{\circ}\text{C}$ , at time  $t$  seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where **A** and **B** are positive constants.

Given that

- the initial temperature of the ethanol was  $18^{\circ}\text{C}$
- after 10 seconds the temperature of the ethanol was  $44^{\circ}\text{C}$

- (a) find a complete equation for the model, giving the values of **A** and **B** to 3 significant figures.  
(4 marks)

(continued on the next page)

9. continued.

Ethanol has a boiling point of approximately  $78^{\circ}\text{C}$

(b) Use this information to evaluate the model.

(2 marks)

(Total for Question 9 is 6 marks)

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10. In this question you must show all stages of your working.

**Solutions relying entirely on calculator technology are not acceptable.**

- (a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4 marks)

- (b) Hence solve, for  $-90^\circ \leq x \leq 180^\circ$ , the equation

$$1 - \cos 3x = \sin^2 x$$

(4 marks)

(Total for Question 10 is 8 marks)

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11. Refer to the diagram for Question 11 in the Diagram Book.

It shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point **P**, shown in the diagram.

- (a) Find the coordinates of **P**  
(2 marks)

- (b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2 marks)

(continued on the next page)

11. continued.

A line  $L$  has equation  $y = ax$ , where  $a$  is a constant.

Given that  $L$  intersects  $y = 2|x + 4| - 5$  at least once,

(c) find the range of possible values of  $a$ , writing your answer in set notation.

(3 marks)

(Total for Question 11 is 7 marks)

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**12. Refer to the diagram for Question 12(a) in the Diagram Book.**

**The curve shown in the diagram has parametric equations**

$$\mathbf{x = 6 \sin t \qquad y = 5 \sin 2t \qquad 0 \leq t \leq \frac{\pi}{2}}$$

**The region  $R$ , shown shaded in the diagram, is bounded by the curve and the  $x$ -axis.**

**(a) (i) Show that the area of  $R$  is given by**

$$\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$$

**(3 marks)**

**(ii) Hence show, by algebraic integration, that the area of  $R$  is exactly 20**

**(3 marks)**

**(continued on the next page)**

**Turn over**

12. continued.

Refer to the diagram for Question 12(b) in the Diagram Book.

Part of the curve is used to model the profile of a small dam, shown **OMNP** in the diagram.

Using the model and given that

- **x** and **y** are in metres
- the vertical wall of the dam is **4.2** metres high
- there is a horizontal walkway of width **MN** along the top of the dam

(b) calculate the width of the walkway.  
(5 marks)

(Total for Question 12 is 11 marks)

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13. The function  $g$  is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where  $k$  is a constant.

(a) Deduce the value of  $k$   
(1 mark)

(b) Prove that

$$g'(x) > 0$$

for all values of  $x$  in the domain of  $g$   
(3 marks)

(c) Find the range of values of  $a$  for which

$$g(a) > 0$$

(2 marks)

(Total for Question 13 is 6 marks)

14. A circle **C** with radius **r**

- lies only in the 1st quadrant
- touches the **x**-axis and touches the **y**-axis

The line **L** has equation  $2x + y = 12$

(a) Show that the **x** coordinates of the points of intersection of **L** with **C** satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3 marks)

Given also that **L** is a tangent to **C**,

(b) find the two possible values of **r**, giving your answers as fully simplified surds.

(4 marks)

(Total for Question 14 is 7 marks)

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15. In this question you must show all stages of your working.

**Solutions relying entirely on calculator technology are not acceptable.**

A geometric series has common ratio  $r$  and first term  $a$

Given  $r \neq 1$  and  $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4 marks)

Given also that  $S_{10}$  is four times  $S_5$

(b) find the exact value of  $r$

(4 marks)

(Total for Question 15 is 8 marks)

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16. Use algebra to prove that the square of any natural number is EITHER a multiple of 3 OR one more than a multiple of 3

(Total for Question 16 is 4 marks)

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**TOTAL FOR PAPER IS 100 MARKS**

**END OF PAPER**

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